

Summer Review Packet – AP Calculus

Directions for this packet: On a separate sheet of paper, show your work for each problem in this packet, and circle your answer. **This is not an option!**

While you will most likely be able to solve or simplify many of these on your own, others may require assistance from outside sources. You are strongly encouraged to:

1. search the internet;
2. work with others (form a study group!); or
3. find other resources to solve these problems (suggestions include possibly checking out a Precalculus book from the high school, looking back over any notes you have kept from the past two or three years, etc.)

All of you have a calculator that either the school has issued to you for this school year, or that you personally own. If you have a school-issued calculator, please keep this for the summer, and bring back at the beginning of the school year in August. If you own your own calculator, you are free to use it as necessary to complete your packet.

You are expected to finish as much of this as possible this summer. We will be spending approximately the first week of school going over any questions you may have. This work will receive multiple grades for correct completion, as well as one quiz grade.

GOOD LUCK!

Mrs. O'Rear

Are You Ready for Calculus?

I) Simplify the following fractions:

1. $\frac{1}{x} + \frac{1}{y} =$

2. $\frac{1}{x} + \frac{1}{x^2} =$

3. $\frac{\frac{1}{x} + 1}{\frac{1}{x}} =$

4. $\frac{\frac{x}{x+y}}{x} =$

5. $\frac{3x}{3x+y} =$

6. $\frac{\frac{1}{x+h} + \frac{1}{x}}{x} =$

II) Factor each expression completely:

7. $x^2 - 16$

8. $x^2 - x - 6$

9. $6x^2 - x - 2$

10. $4x^3 - 19x^2 - 5x$

11. $x^2 + 9$

12. $x^4 - 13x^2 - 30$

13. $x^3 + 27$

14. $x^3 - 8$

15. $(2x - 3)^3(x + 1) + (x - 3)(2x - 3)^2$

16. $(3x - 2)^{-4}(x + 3) + (x + 3)^2(3x - 2)^{-3}$

III) Solve the following equations for x :

17. $x^2 + 5x - 24 = 0$

18. $x^2 - 9 = 5$

19. $x^3 - x^2 - x = 0$

20. $3x^2 - 5x - 2 = 0$

21. $x^2 - 4x = 0$

22. $(x-1)(x^2 - 11x + 30) = 0$

23. $\sqrt{x} + 1 = 41$

24. $\frac{y}{x+1} = \frac{z}{x}$

25. $\sqrt[3]{x+1} - 4 = -1$

26. $\log_5 x = 2$

27. $x^{-2} = \frac{1}{9}$

28. $\log_8 x = \frac{-1}{3}$

29. $2\sqrt{x} = x - 3$

30. $\log_3 \frac{1}{81} = x$

31. $\frac{8+x}{x} - 5 = 0$

32. $x^{-1} = -3$

33. $x^{\frac{4}{3}} = 81$

34. $x^2 + x - 1 = 0$

35. $\sqrt{x} = \sqrt[3]{x}$

36. $(x-3)^2 + 9 = 25$

37. $3x^2 - 6x - 24 = 0$

38. $\frac{2x-1}{(x+2)(x^2+3)} = 0$

39. $x^3 - 2x^2 - 5x + 6 = 0$

40. $\frac{x}{x+2} - \frac{2}{2x-1} = \frac{1}{5}$

III) Solve each equation for x : (continued)

41. $xy - z = z + wx - 3x$

42. $\frac{4x}{x^2 + 3} - 1 = 0$

43. $(2x - 1)^2(x - 5)^2 + (2x - 1)^3(x - 5) = 0$

IV) Show that each equation is true by simplifying the following:

44. $\frac{1}{\sqrt{x} + \sqrt{y}} = \frac{\sqrt{x} - \sqrt{y}}{x - y}$

45. $\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$

46. $\frac{x - y}{\sqrt{x} + \sqrt{y}} = \sqrt{x} - \sqrt{y}$

47. $\frac{\sqrt{x+4} - 2}{x} = \frac{1}{\sqrt{x+4} + 2}$

V) Which of the following expressions equals $\log 4$?

48. $2\log 2$

49. $\frac{\log 8}{\log 2}$

50. $\log 8 - \log 2$

51. $\log 4 + \log 1$

52. $\log 4 \bullet \log 1$

53. $(\log 2)^2$

VI) Write an equation of a line based on the given information:

54. Find the equation of the line that has a slope of 5 and passes through the point (3, -4).

55. Find the equation of the line that passes through the points (4, 1) and (3, -2).

56. Find the equation of the line that passes through the point (-2, 1) and is parallel to the line $4x + 2y = -1$

57. Find the equation of the line that has a slope of zero and passes through the point (-5, 1).

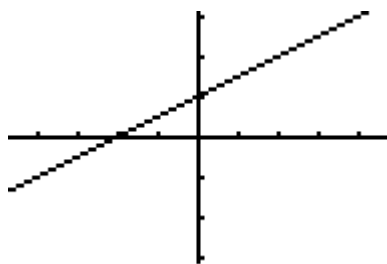
58. Find the equation of the line that passes through the origin and is perpendicular to the line $3x + 4y = -7$
59. Find the equation of the line that has an undefined slope and passes through the point $(4, -5)$.
60. Find the equation of the line that has an x -intercept of 5 and a y -intercept of 3.
61. Find the equation of the vertical line that passes through the point $(3, 2)$.
62. Find the equation of the horizontal line that passes through the point $(1, -5)$.

VII) Find the intercepts:

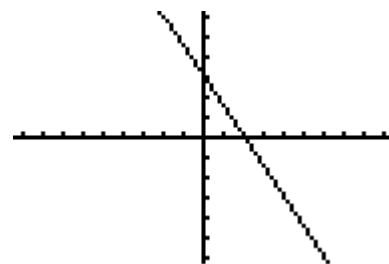
63. Find the x and y intercepts of the line that passes through the point $(3, -5)$ and is perpendicular to the line $x - 3y = -2$.
64. Find the x and y intercepts of $\frac{x^2}{4} - \frac{y^2}{9} = 1$
65. Find the x and y intercepts of $\frac{y^2}{4} + \frac{x^2}{16} = 1$
66. Find the x and y intercepts of $x^2 + y^2 = 9$

VIII) Write the equation of the following graphs:

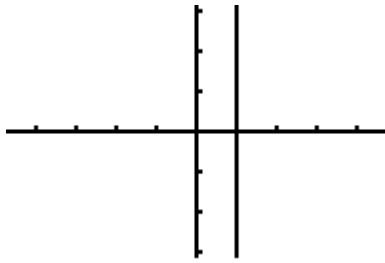
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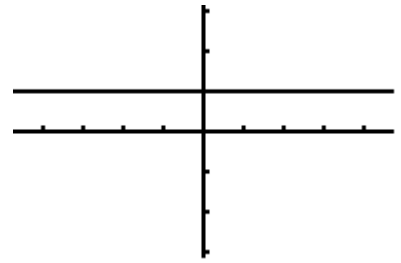
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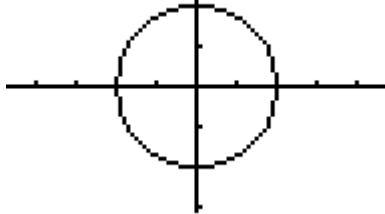
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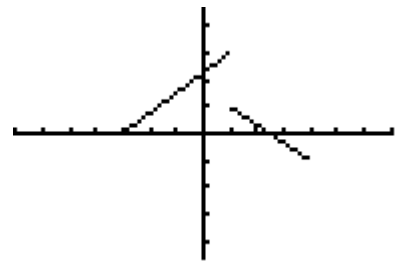
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72.



71.



IX) Given the slope, sketch the following lines:

73. Sketch a line with a slope of 2.

74. Sketch a line with a slope of $\frac{1}{2}$.

75. Sketch a line with a slope of -2.

76. Sketch a line with a slope of $-\frac{1}{2}$.

X) Sketch the following graphs:

77. $y = 3x + 1$

78. $y = x^2$

79. $y = |x|$

80. $y = x^3$

81. $x = 3$

82. $y = -4$

83. $y = \ln x$

84. $y = \sqrt{x}$

85. $y = \frac{1}{x}$

86. $y = \frac{1}{x^2}$

87. $y = |x + 1|$

88. $y = x^2 + 2x - 3$

89. $y = x^3 + 1$

90. $y = (x + 1)^2$

91. $y = -x^2 + 1$

92. $y = (x + 1)^{-\frac{1}{2}}$

93. $y = x^{\frac{2}{3}}$

94. $y = (x + 1)^{\frac{1}{3}}$

95. $y = \log x$

96. $y = e^x$

97. $y = -\frac{1}{x}$

98. $x^2 + y^2 = 25$

99. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

100. $\frac{x^2}{16} - \frac{y^2}{4} = 1$

101. $y = \begin{cases} -2 & \text{for } x < -1 \\ x & \text{for } -1 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$

102. $y = \begin{cases} x + 1 & \text{for } x < 1 \\ x^2 & \text{for } x \geq 1 \end{cases}$

XI) Rewrite the following functions without absolute values:

103. $y = |x|$

104. $y = |x - 1|$

105. $y = |x + 3| + 1$

106. $y = \frac{|x| - 2}{x - 2}$

XII) Find the domain and range of each function:

107. $y = x - 1$

108. $y = \frac{1}{x}$

109. $y = \frac{1}{x^2 + 1}$

110. $y = \frac{1}{\sqrt{x^2 + x + 1}}$

111. $y = \sqrt{x - 4}$

112. $y = |x - 1| + 2$

113. $y = \log x$

114. $y = 2^x$

115. $y = \sqrt{x^2 - 3x - 4}$

116. $y = \begin{cases} x + 3 & \text{for } x < 2 \\ x^2 & \text{for } x > 2 \end{cases}$

XIII) Find the inverse of each function:

117. $f(x) = x + 3$

118. $f(x) = \sqrt{x}$

119. $f(x) = \frac{x}{x + 2}$

120. $f(x) = 3^x$

XIV) Find the compositions of the function if:

$$f(x) = x^3 + 1, \quad g(x) = x^2 - 2, \quad \text{and} \quad h(x) = x + 3$$

121. $f(2) =$

122. $f(h(x)) =$

123. $f(g(2)) =$

124. $g(g(x)) =$

125. $g(f(x)) =$

126. $h(g(x)) =$

127. $h(x^2) =$

128. $f(x + h) =$

129. $h(p) =$

130. $\frac{g(t + h) - g(t)}{h}$

XV) Solve the simultaneous equations:

131.
$$\begin{aligned} 2x + 3y &= 8 \\ x + 2y &= 5 \end{aligned}$$

132.
$$\begin{aligned} y &= x^2 + 2x + 9 \\ 7x + y &= 19 \end{aligned}$$

XVI) Intersection of curves:

133. Find the point of intersection between the lines $y = x + 1$ **and** $3y - x = 5$.
134. Find the points of intersection between the line $y = x + 7$ **and the curve** $y = x^2 + 2x + 5$. Then sketch the area between the graphs.

XVII) What do the following mean if:

135. a graph is in the first quadrant
136. $f(2) = 5$
137. an expression is a function
138. a zero of a function is 4
139. y is directly proportional to x (give an example)
140. the coefficient of the third term is 5 (give an example)
141. a function only has one root
142. a function is a polynomial
143. two triangles are similar

XVIII) What are the following formulas:

144. quadratic formula
145. Pythagorean theorem
146. the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length x
147. the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of x
148. the volume of a sphere
149. the volume of a cylinder

- 150. the volume of a cone
- 151. the volume of a box with a square base
- 152. the surface area of a sphere
- 153. the surface area of a cylinder with no top
- 154. the area of a triangle
- 155. the area of a trapezoid
- 156. the cross section through the center of a sphere
- 157. the volume of a prism that has an equilateral triangle with side length of x and height of length y

XIX) Solve by using similar triangles:

- 158. A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow?
- 159. Water is dripping out of a conical figure that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water?

XX) Describe the symmetry of the following functions:

160. $y = x^2 + 1$

161. $y = x^3 + 1$

162. $y = |x|$

163. $y = x^3 + x$

164. $y = \sin x$

165. $y = \cos x$

XXI) Find the equations for the horizontal and vertical asymptotes of each function:

166. $y = \frac{1}{x-1}$

167. $y = \frac{x}{\sqrt{x^2-9}}$

168. $y = \frac{x^3}{x^3 - 1}$

169. $y = \frac{x}{\sqrt{x^2 + 1}}$

XXII) Write the following without sigma notation:

170. $\sum_{n=1}^4 2n + 1$

171. $\sum_{n=2}^7 2$

XXIII) Exponent rules: Which of the following are true?

172. $x^0 = 1$

173. $x^{-2} = \frac{1}{x^2}$

174. $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

175. $x^5 \cdot x^3 = x^{15}$

176. $x^5 \cdot y^5 = (xy)^5$

177. $(x^3)^5 = x^8$

178. $x^{5-w} = \frac{x^5}{x^w}$

179. $x^{t+s} = (x^t)^s$

180. $\sqrt{\frac{9}{4}} = \frac{3}{2}$

181. $(4x)^{\frac{1}{2}} = 2x$

182. $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$

183. $\sqrt{x^2} = x$

184. $\sqrt{x^2 - 1} = x - 1$

185. $x^{\frac{4}{3}} = \sqrt[4]{x^3}$

186. $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$

187. $x^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{x^2}}$

188. $e^{\ln x^2} = x^2$

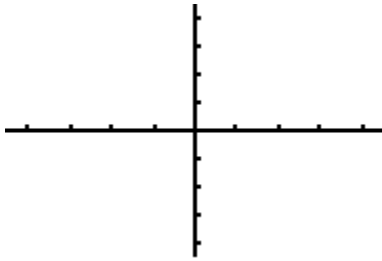
189. $\ln e^3 = 3$

190. $e^{2\ln 2 - \ln 4} = \frac{4}{5}$

191. $\ln x^2 = (\ln x)^2$

XXIV) Using the graphing calculator:

192. Graph $y = .1x^3 + 2x^2 - x - 3$ on the x - y plane below:



193. Find the roots of the equation above.

194. Find the points of intersection for the graphs $y = x^3 + x - 3$ and $y = 2x + 4$.

195. Find the maximum value for the graph $f(x) = -x^4 + x - 4$.

196. $f(x) = -x^4 + x - 4$. On what intervals for x is f increasing?

XXV) What are the following trigonometric identities:

197. $\sec x =$

198. $\csc x =$

199. $\tan x =$

200. $\cot x =$

201. $\cos^2 x - 1 =$

202. $\sec^2 x - 1 =$

203. $\cot^2 x + 1 =$

204. (half angle) $\cos^2 x =$

205. (half angle) $\sin^2 x =$

206. (double angle) $\sin 2x =$

207. (double angle) $\cos 2x =$

208. $\sin(a + b) =$

209. $\cos(a + b) =$

XXVI) Find the period of the following functions:

210. $y = 4\sin(2x) + 1$

211. $y = 3\tan(\pi x)$

212. $y = \cos^2(x)$

213. $y = -3\sec(-6x) + 2$

XXVII) Evaluate the following expressions:

214. $\sin\left(\frac{\pi}{6}\right) =$

215. $\cos^{-1}\frac{\sqrt{3}}{2} =$

216. $\tan(210^\circ) =$

217. $\cos(0) =$

218. $\cos(45^\circ) =$

219. $\csc\left(\frac{-5\pi}{6}\right) =$

220. $\sec(180^\circ) =$

221. $\cot\left(\frac{-2\pi}{2}\right) =$

222. $\sin^{-1}\left(\frac{1}{2}\right) =$

223. $\tan\left(\frac{\pi}{2}\right) =$

224. $\sin^2\left(\frac{5\pi}{6}\right) =$

225. $\cot\left(\frac{2\pi}{3}\right) =$

226. $\sin\left(\frac{\pi}{2}\right) =$

227. $\cot^{-1}(-1) =$

228. $\sec\left(\frac{3}{4}\right) =$

229. $\tan^{-1}(-1) =$

230. $\sec(180^\circ) =$

231. $\sec^2\left(\frac{-11\pi}{6}\right) =$

232. $\sin(\csc^{-1}(-2)) =$

233. $\sec^{-1}(\sec(-30^\circ)) =$

XXVIII) Sketch one period of the following trigonometric graphs:

234. $y = \sin x$

235. $y = \cos x$

236. $y = \tan x$

237. $y = \sec x$

238. $y = \csc x$

239. $y = \cot x$

240. $y = \sin(2x)$

241. $y = 3\cos(4x) - 1$

242. $y = \cos^2(x)$

243. $y = |\sin x|$

244. $y = \cos(\pi x)$

245. $y = \cos(-\pi x)$

XXIX) Solve the following trigonometric equations for the given domain:

246. $\sin x = \cos x$ on $[0, 2\pi]$

247. $\sin 2x = \frac{1}{2}$ on $[0, 2\pi]$

248. $\sin 3x = \frac{\sqrt{3}}{2}$ for all x

249. $\cos 4x = -\frac{1}{2}$ for all x

250. $\tan 2x = 0$ on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

251. $\sec \pi x = 2$ for all x

252. $\cot x = 0$ on $[0, \pi]$

253. $\cot ax = 0$ for all x

254. $2 \sin x \cos x = 0$ for all x

255. $\sin^2 x + \sin x = 0$ on $[0, 2\pi]$